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7/25/68

DEVELOPMENT OF CRITERIA FOR THE CONSTRUCTION OF THE MOST FAVORABLE NETWORK FOR SHORT-RUN MAINTENANCE PROJECTS

A THESIS

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DEVELOPMENT OF CRITERIA FOR THE CONSTRUCTION

OF THE MOST FAVORABLE NETWORK FOR

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Approved:

Chairman

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This thesis is dedicated to my parents, Mr. and Mrs. Alfredo Pretoni, for their foresight, guidance, and unselfish sacrifice without which the author's graduate program would not have become a reality. I also extend my deepest gratitude to my wife, Mag, who always encouraged me toward higher goals in life, and to Eduardo, whose smile and first words of his life were a constant source of inspiration during the formulation of this thesis.

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SUMMARY

One of the possible improvements in the planning and scheduling of short-run maintenance projects is the application of the PERT-CPM techniques. However, some difficulties have arisen when attempts have been made to apply these techniques. The main problem is the drawing of the network, where the identification of the activities is very difficult because it is not known exactly what part of the system has failed.

By using the concepts of reliability theory and the principles of PERT-CPM theory, one approach was developed for the design of networks which represent, mainly in terms of re-scheduling costs, the Most Favorable Network (MFN) for the maintenance project. Some concepts of decision theory were also used for determining an index that ranks all the possible networks of the project. A guide for filling the needs of management for planning and scheduling short-run maintenance projects is presented.

LIST OF ABBREVIATIONS

BA ....Basic Activity

CACP ....Complementary Activity in the Close Up Phase

CAOP ....Complementary Activity in the Opening Phase

CARP ....Complementary Activity in the Repair Phase

CP ....Close Up Phase in the Maintenance Project

DA ....Dangerous Activity

GA ....Good Activity

IA ....Image Activity

MA ....Maintenance Activity

MAOR ....Middle Activity Between the Repair and the Close Up Phase

MFN ....The Most Favorable Network

OP ....Opening Phase in the Maintenance Project

RP ....Repair Phase in the Maintenance Project

TNP ....Theoretical Network of the Project

CHAPTER I

INTRODUCTION

General

The need for a better maintenance organization, adequate control, and effective planning and scheduling has been emphasized by several factors such as increased mechanization, complexity of the equipment, parts and supplies inventories. With the many factors contributing to increased costs of maintenance, its importance is becoming greater in every industry. Thus, management has begun to focus more and more attention on this function.

Particularly, PERT-CPM techniques have been used by management for planning and scheduling maintenance operations. These techniques have been applied with good results on preventive maintenance and on overhaul corrective maintenance projects. However, up until now PERT-CPM techniques have not been applied with good results on short-run maintenance projects because:

(1) It has been difficult to draw a correct and complete network for the projects, since it is not exactly known what the activities are going to be.

(2) If the network has already been drawn, the re-planning and re-scheduling of such networks tend to become so frequent that the application of PERT-CPM techniques will not yield economic and efficient results. Generally the proper network is reached only when the system under consideration has been completely repaired. Improved implementation believed procedures are essential if the PERT-CPM approach is to realize its full potential for short-run maintenance projects.

The concepts and applications of reliability theory have also become increasingly important nowadays. The growth of increasingly complex military and industrial equipment has focused attention on the reliability of such systems. The automatic control of large and expensive industrial processes has served to emphasize the importance of reliability as a performance measure comparable to such other requirements as efficiency, speed and accuracy. The importance of the fact that a system or device is not operating is magnified as the size and cost of such operations increase.

Objective and Purpose

Looking at the problems that come from PERT-CPM applications on short-run maintenance projects and the availability of the concepts of reliability theory, the objective of this research is to develop an approach for the design of a network that represents the best pre-determined plan for the maintenance project under consideration. This network will be referred to as **THE MOST FAVORABLE NETWORK** of the project (**MFN**).

Efforts will be concentrated on planning the maintenance actions for corrective maintenance in which it is not known what part has failed. Suggestions will be made for extensions to preventive maintenance, in which some non-failed parts are replaced.

Motivation for Research

The assumptions behind these objectives and the motivation for this research are the beliefs that:

(1) The solution of the scheduling and planning problems in the maintenance area is important and sometimes vital for an enterprise and may represent significant financial savings.

(2) There exists a high probability that the approach that is going to be established can be successfully applied to real short-run maintenance projects.

(3) There is a good possibility of extending the

approach to other maintenance fields with positive results.

(4) The application of the approach will not be limited to a particular piece of equipment or a particular enterprise but will include any system that has the maintenance function associated with it.

CHAPTER II

APPLICABLE BASIC CONCEPTS OF PERT-CPM AND RELIABILITY

Literature Survey

A search of the literature survey shows that, although much has been written about the two fields, reliability theory and PERT-CPM, little effort has been made to relate them to each other, and nothing has been done in respect to the application of both these techniques on maintenance problems, particularly on short-run maintenance projects.

In the first case, H. G. Romig (29) has presented a paper in which he discusses possible application of PERT-PEP (Program Evaluation and Review Technique - Program Evaluation Procedure) systems on research, development, planning and production of complex products. It is recognized that controls have been placed on the development, planning and production of both commercial and military equipment. What is desirable are reliability key points with adequate monitoring and policing. The PERT-PEP systems are applied particularly in determining these key points for all phases of the product, in such a way that it may be produced efficiently.

As stated above, it appears that nothing has been done toward the simultaneous application of both techniques on maintenance problems. However, some problems in the maintenance field have been studied by applying these techniques separately. ARINC Research Corporation (2), W. G. Ireson (17), D. W. Jorgenson (18), L. C. Morrow (24), R. H. Myers (25) and E. L. Welker (36) have discussed the value of reliability concepts for scheduling preventive maintenance.

On the other hand, there are some articles that present application of PERT-CPM concepts and a very few that discuss the possibility of existence of some part that has failed and has not been considered in drawing the network.

Eric R. Reeves (28) in a study of plant turnaround points out that

> ...when the planner has all the necessary data he should prepare the first overall diagram. I use the words " first overall diagram " since on projects of the magnitude of a plant turnaround the final network rarely resembles the first draft. Refinement of the plan is an extremely important factor but, unfortunately, often an overlooked phase of CPM.

In another paper, J. I. Vander Raadt (27) discusses the application of CPP (Critical Path Planning) to process unit shutdown maintenance. He makes the following observation:

> ...a difference between a construction project and a shutdown maintenance project is that in planning the shutdown we cannot define the full scope of the work in advance. Thus, the plan must make it possible to absorb additional work without panicking or making unnecessary expenditures.

Furthermore, Vander Raadt makes some suggestions to decrease the impact of the unforeseen factor:

> At our plant we have a permanent work schedule for each unit that is used for all shutdowns. This includes all jobs that must be done at regular intervals. Thus, it is only necessary to supplement the routine schedule with a list of the additional work needed, a supplementary work list that is normally done by the foreman.

Kenneth Brooks (8) applies CPM to turnarounds for Cities Services. In this article the problem of unknown parts failed is solved " from experience and records of past turnarounds, making an educated guess. "

One soon discovers that the problem of forecasting failed parts has not been solved or it has not been solved satisfactorily. Thus, by applying reliability concepts and PERT-CPM theory to the short-run maintenance project, the writer seeks to give a scientific approach to designing the original network in such a manner as to make it closer to the real network. The ideas presented in the articles discussed here will also be utilized.

Furthermore, the concepts and procedures proposed in this research will have practical applications for solving network modelling problems in any company faced with problems of planning short-run maintenance projects.

PERT-CPM Concepts

Basic Characteristics of Network-Based Project Management

Within the last decade, a powerful but simple approach to planning, scheduling and controlling large and complex projects was added to the tools available for decision making. It is the CPM (Critical Path Method) and/or PERT (Program Evaluation and Review Technique) approach. The basic approach involves network representation of the project, which may be defined as a set of interrelated activities whose purpose is to attain a specific objective by a given method. Each project has several characteristics that are essential for analysis by PERT-CPM. These include the following:

(1) The project consists of a well defined collection of activities which, when completed, mark the end of the project.

(2) The internal structure of an activity is independent of every other activity.

(3) The activities are ordered, i.e., they must be performed according to a set of precedence relationships. If the goal of one activity must be achieved before the next one can start, the activities are precedence related. A typical project involves a considerable degree of precedence relationship among its activities. The short-run maintenance projects particularly fall into this category.

Network Representation of a Project

The basis of both PERT and CPM is the project network diagram. Two systems of networking which are most widely used are the following:

(1) Activities-on-Arrows System (A-O-A)

(2) Activities-on-Nodes System (A-O-N).

While the basic concepts underlying both these systems are essentially the same, significant differences exist in the mode of representation.

Activities-on-Arrow System (A-O-A): In this system each activity in the project is defined and represented by an appropriately labeled arrow. Each arrow is placed in the network in proper relation to the other arrows. That is, activities can occur in relation to other activities in three ways: one activity can precede another; activities can be done concurrently; one activity can follow another. At certain points in the network dotted arrows may be inserted to indicate constraints. These dotted arrows are named dummies. They do not represent actual activities, but are put in the diagram to complete the technological work sequence by indicating the interrelationship between activities. Dummies are also used to prevent arrows from having common beginning and end points. Figure 1 lists some configurations which appear frequently in networks together with the relationship they define among activities.

Activities-on-Node System (A-O-N): In this system the nodes or circles are used to represent activities in the project, i. e., each circle in a network corresponds to an activity in the project. Arrows connecting circles are used to depict the relationship among the activities. The A-O-N system was selected as being best suited to the representation of the networks of the research done herein. Figure 1 provides a comparison of the two networking systems, as the same situation is graphically represented in both systems. An examination of Figure 1 reveals that the dummy arrows that are often needed in the arrow notation (A-O-A) have no analog in a circle notation (A-O-N) network.

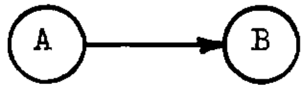
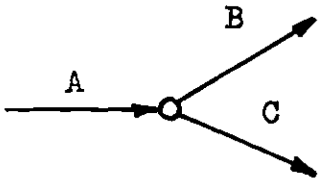
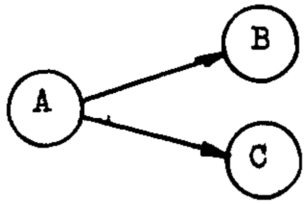
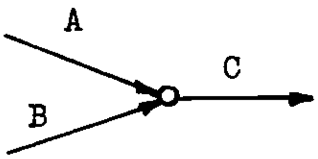
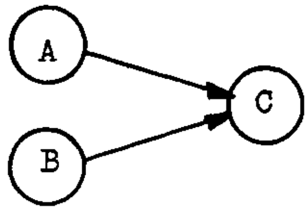
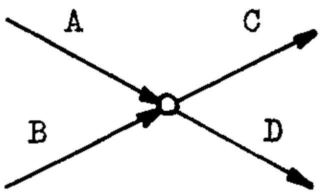
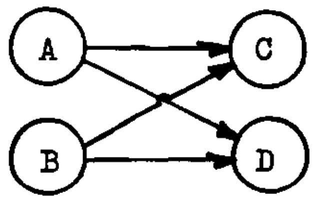
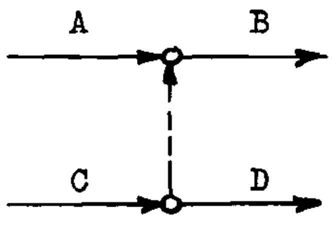
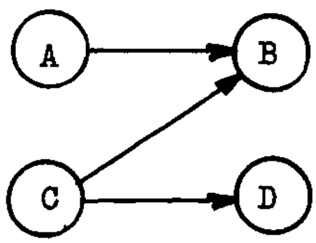
A-O-A Diagram	A-O-N Diagram	Logic
		Activity B can begin only after activity A is completed.
		Neither activity B nor C can start before activity A is completed, but B and C can be performed concurrently.
		Activity C can begin only after both activities A and B are completed.
		Neither activity C nor D can begin until both A and B are completed, but C can be started independent of D or vice-versa.
		Activity B cannot begin until both A and C are completed, but D can start after only C is completed. The dotted arrow is a dummy activity.

Figure 1. Common Network Diagrams

Any logic restriction can be displayed in circle notation without the use of dummy activities. However, dummies can be used as a convenience in the A-O-N system.

The Phases of Network-Based Project Management

The purpose of this section is to introduce the reader to the three phases of project management. These three phases are Planning, Scheduling and Control.

The Planning Phase. The most essential factor to the success of project management is planning. The purpose of planning is to establish the end objectives and to define the activities and their relationships, assuring in this manner that the project progress has specific direction.

PERT-CPM planning begins with an analysis of the project objective and a clear definition of the activities. The level of detail of the analysis depends largely on the purpose of the plan and on the planner's ability to identify individual activities.

The output from the planning phase is a network which represents the sequences of activities that must be followed during the project. Based on this network, all the other phases will be developed.

This research will concentrate on the design of this network with the objective being to make possible the best performance for all phases of the PERT-CPM systems for short-run maintenance projects.

The Scheduling Phase. Once the planning phase of a project network model has been completed, work can begin on converting the plan into a workable schedule which can be used as a guide for implementing a project. The scheduling phase is concerned with establishing start and completion times for each activity. In this phase the project is examined in the light of its restrictions and techniques for resource leveling or allocating limited resources may be used. These approaches may be strengthened by including in them the consideration of the alternative activity durations associated with alternative resource levels.

In this phase the earliest start, latest start, earliest completion, and the latest completion of each activity in the project is calculated. The constraints of the project are also considered, and the so-called critical path is determined. The activities lying on that path determine the minimum project length and require special attention in the evaluation of the availability of certain key resources.

The Control Phase. In order to control a project, a project manager needs to take corrective action where significant deviations in actual progress and costs from planned progress and costs begin to appear. He takes the schedule as a desired level of performance and compares with that schedule the actual performance. The difference, if any, leads to control measure to eliminate the difference. A new schedule condition for the next control point is identified and the control cycle continues. In other words, the following questions are answered:

(1) Will the project be completed on time?

(2) Will the final cost be within the estimated amount?

(3) Are the required performance criteria correct?

Reliability Theory

Introduction and Some Concepts

In the material that follows a brief description is given of what is meant by the term reliability and by some terms that are related to reliability theory. The author will present the basic definitions and relations between the hazard rate of failure, failure density function, failure distribution function and reliability function for catastrophic failure model.

Reliability has been defined as "the probability of a device or system performing adequately for the period of time intended under the operating conditions encountered" (2).

The random variable t^* is defined as the time to failure of the item in question. Thus, the probability of failure as a function of time is given as

$$P\ (t^* \leq t) = F(t) \tag{1}$$

which is the definition of the failure distribution function.

The reliability function, R(t), which is the probability that the item will not fail in t time periods, is defined in terms of $F(t)$, as being

$$R(t) = 1 - F(t) = P\ (t^* \geq t) \tag{2}$$

The failure density function is given by

$$dF(t)/dt = f(t) \tag{3}$$

From equation (1), the probability of failure in a time interval Δt is defined as being

$$P\ (t < t^* \leq t + \Delta t) = F(t + \Delta t) - F(t) \tag{4}$$

Equation (4) can be written in terms of probability of survival up to time t, $R(t)$, and the conditional probability of failure in the interval $t < t^{*} \leq t + \Delta t$, given survival up to time t, as

$$P\,(t < t^{*} \leq t + \Delta t) = R(t)\,P\,(t < t^{*} \leq t + \Delta t \mid t^{*} > t) \qquad (5)$$

Combining the equations (4) and (5), one obtains

$$P\,(t < t^{*} \leq t + \Delta t) \mid t^{*} > t) = \qquad (6)$$

$$= \frac{F(t + \Delta t) - F(t)}{R(t)}$$

Dividing both sides of (6) by Δt and taking the limit as $\Delta t \rightarrow 0$ yields

$$\frac{dF(t)/dt}{R(t)} = \frac{f(t)}{R(t)} = z(t) \qquad (7)$$

The equation (7) shows that $z(t)$ is the time rate of change of the conditional probability to failure. This is called <u>hazard rate of failure</u>, concluding in such a manner the relationship among $f(t)$, $F(t)$, $R(t)$, and $z(t)$.

The shape of the reliability function and of the hazard rate of failure curves for the most frequently encountered failure distributions are shown in the Figure 2.

<table>
<tr><th></th><th>Rel. Function, R(t)</th><th>Hazard Rate , z(t)</th></tr>
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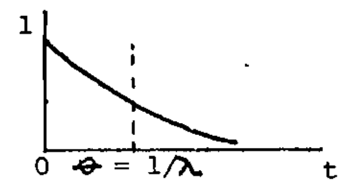
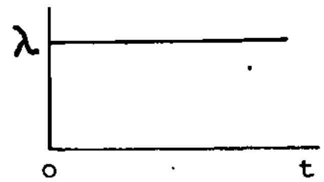
<tr><td>
</td><td>
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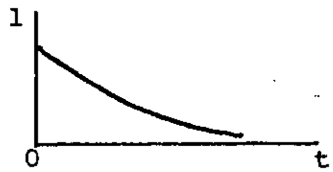
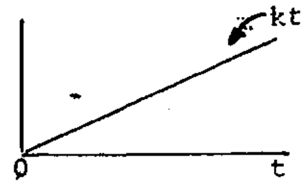
<tr><td>
</td><td>
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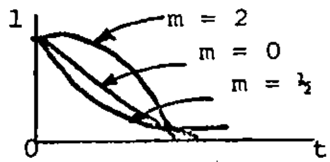
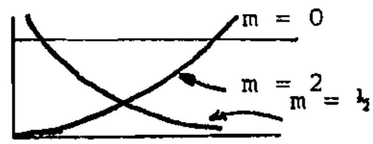
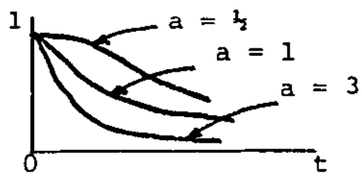
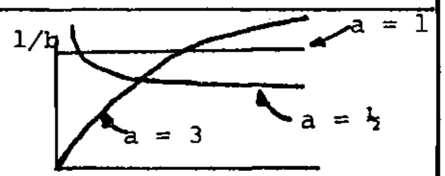
<tr><td>
</td><td>
</td></tr>
<tr><td rowspan="2">GAMMA</td><td>$\frac{1}{(a-1)!b^a}\int_t^\infty \xi^{a-1}e^{-\xi/b}\,d\xi$</td><td>$\frac{t^{a-1}e^{-t/b}}{\int_t^\infty \xi^{a-1}e^{-\xi/b}\,d\xi}$</td></tr>
<tr><td>
</td><td>
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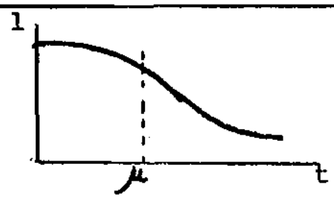
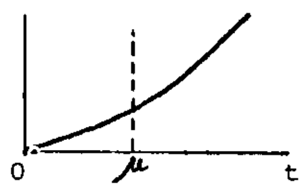
<tr><td>
</td><td>
</td></tr>
</table>

Fig. 2. Shapes of Common R(t) and z(t).

Maintainability: Some Concepts

This section will present some concepts about maintainability theory and its basic elements, since they will be used in the scheduling phase of Network-Based Project Management of maintenance projects.

Basic Definitions. Maintainabilityis defined as being "the probability that a failed system is restored to a operable conditions in a specific down-time, when the maintenance is performed under stated conditions" (2).

Down-Time is defined as being "the interval of time during which the system is not in acceptable operating conditions, i. e., the time from the initiation of a complaint or most routine maintenance actions to restore the system to satisfactory conditions" (2).

Some Comments About Maintainability Prediction. The system hardware factors and the human factors have considerable effect on maintenance time. The total length of time that a system is down for active maintenance varies statistically from one failure to another. Furthermore, it also varies from one repetition to the next of a given failure type and the corresponding repair cycle.

Analysis of experimental data on down-times shows that the observed range of variables can usually be fitted by a statistical distribution such as the Normal, Gamma, Exponential and Logarithm Normal (2), (3).

For each type of equipment there will be a specific distribution of down-time; this is more developed for electronic equipment than for a mechanical one. However, even for mechanical equipment it is possible to determine a down-time distribution. This must be done to achieve an optimal point either in equipment availability or in the total cost of operation and maintenance.

CHAPTER III

THEORETICAL NETWORK OF A PROJECT

Introduction

The maintenance project manager needs a plan by which he undertakes a short-run maintenance project. This plan will be selected from among all the possible starting plans for this project. This chapter presents a procedure for collecting, classifying and displaying the total set of possibilities for the components of the project.

Definition of the Theoretical Network of a Project

For the purpose of investigating the project alternatives, any maintenance project may be divided into three different phases. They include the following:

(1) Opening Phase (OP): In this phase all parts of the system involved in the maintenance project must be processed in whatever manner required to prepare the possible failed parts for inspection and repair. Hence, all activities required for this preparation should be included.

(2) Inspection and Repair Phase (RP): In this phase all parts of the system are inspected and repaired. Consequently, all the activities required to perform the repair and inspection should be included.

(3) Close Up Phase (CP): In this phase all parts of the system not restored to operating condition in the repair phase, i.e., parts not subject to failure, are reassembled into the system such that the system can start working. Thus, all the activities required to make the system operational again should be included.

The network which contains these three phases completely detailed and connected to each other will be called the Theoretical Network of the Project (TNP).

A procedure for development of the TNP is essential. Some important "key points" have to be defined and determined in these phases; there will be some correspondence among them in such a way that they will be the steppingstones for the development of the Most Favorable Network of the Project (MFN). This procedure will be considered in the following section.

Procedure for Development of the TNP

In drawing the TNP we must have in mind the following steps:

Step 1 : Preserve all rules for the drawing of a common network in the A-O-N system as well as in the A-O-A system. These rules are presented in the Figure 1.

Step 2 : In the OP, every activity is associated with one or more parts or sub-systems that may fail.

Step 3 : Each activity in the OP will be concerned only with reaching a state in which one or more parts are ready for inspection.

Step 4 : In the CP, every activity is associated with one or more parts or sub-systems that may fail.

Step 5 : Each activity in the CP will be concerned only with transforming the system from a state in which the failed part has been restored to a state in which normal operation can be resumed.

Step 6 : Normally, each activity in the OP will have a corresponding activity in the CP. For example: dismantling a generator in the OP and assembling it in the CP. Sometimes, it will be necessary to include different activities in the OP and CP phases and the correspondence will no longer exist.

Step 7 : The activities in the OP which have corresponding activities in the CP will be called Basic Activities (BA), and the corresponding activities in the CP will be called Image Activities (IA). The other activities in OP and CP will be called Complementary Activities in OP (CAOP) and Complementary Activities in CP (CACP). For these the correspondence does not exist.

Step 8 : In the RP, one activity will represent the maintenance operation for each part of the system that has to be repaired. This activity is referred to as a Maintenance Activity (MA).

Step 9 : Basically the precedent and subsequent activities of a maintenance activity will be its correspondent BA and IA.

Step 10 : In the RP sometimes there will be a network associated with each part, instead of the MA alone. In this case the activities different from the MA itself are referred to as Complementary Activities in RP (CARP), and the relations of precedence among them and their corresponding BA and IA shall be analysed for each case studied.

Step 11 : There will exist two additional activities between the OP and the RP and between the RP and OP. The first one is the Middle Activity Opening-Repair Phases (MAOR), and

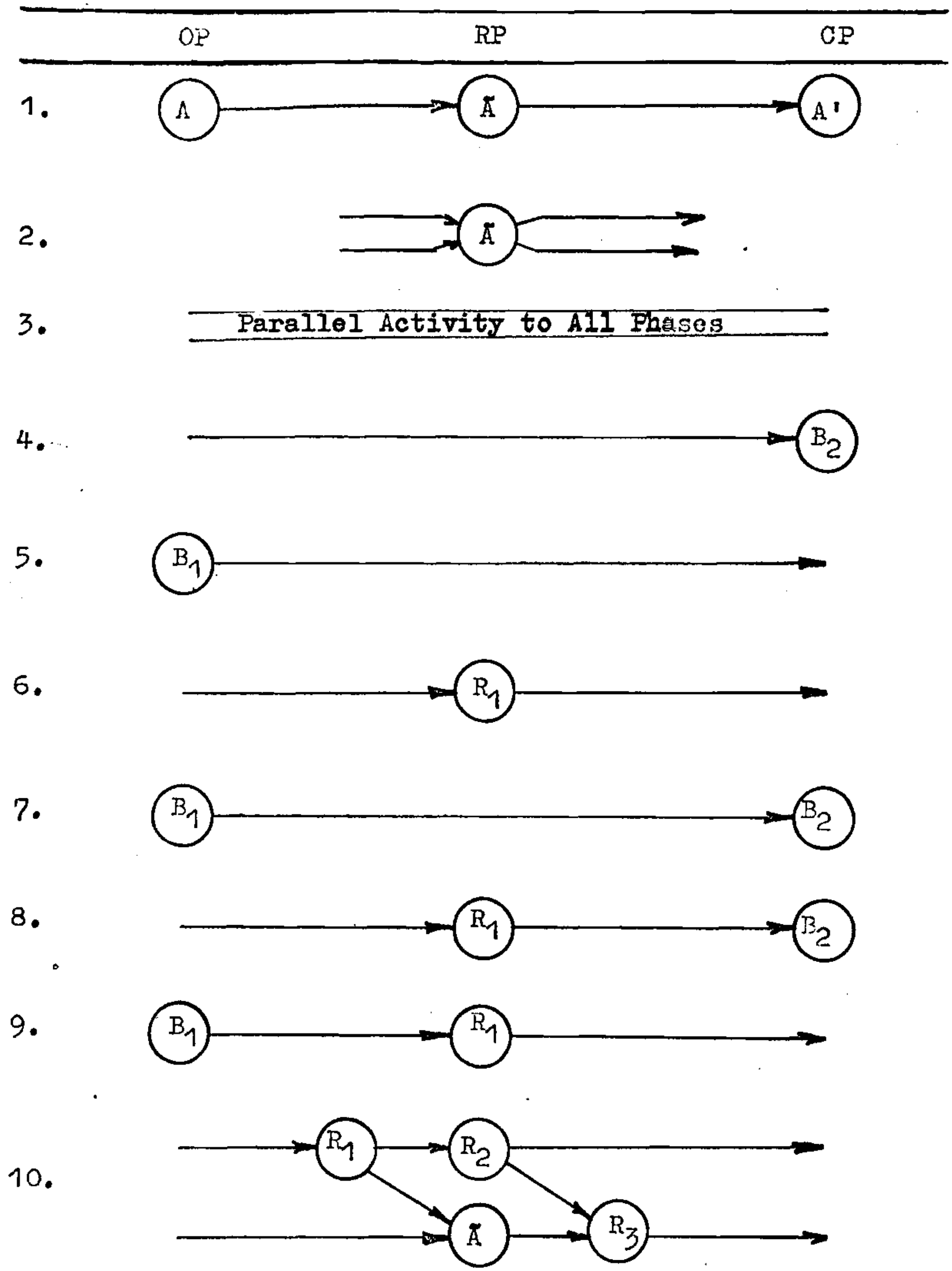


Figure 3. Possible Cases in Drawing the TNP

the second one the Middle Activity Repair-Close Up Phases (MARC). These could be used to disconnect the three phases one from another, since sometimes it will be useful to know the networks of the three phases separately or to eliminate some parallelism problems. These two cases will be discussed in drawing up the Most Favorable Network.

Step 12 : In Figure 3 the possible cases that may exist in drawing the TNP are shown and must be considered carefully. In every case, the detail to which the network is drawn must be such that the precedence relationships among all the activities have no ambiguity. For example, if one has the case (3) in Figure 3, in which the activity is parallel to the three phases, this activity could be broken into three parts so that each part belongs to one phase. This operation could be done using the middle activities.

Step 13 : If the existence of the middle activities is required, then the MAOR is an end point for OP and a starting point for the RP; the MARC is an end point for the RP and a starting point for the CP.

Example of a TNP

To illustrate these ideas and to make easier the exposition of the next chapters let us consider a special network that will be used during all this research. Let us network that will be used during all this research. Let us suppose that this network presents the characteristics of Figure 4, in which the following is true:

(1) The network is represented by the A-O-N system.

(2) A_1 is the activity START.

(3) A_2 is the activity END.

(4) B, F, H, I, K, and L are basic activities.

(5) B', F', H', I', K', and L' are image activities.

(6) C_1, D_1, E_1, G_1, and J_1 are the CAOP.

(7) C_2, D_2, E_2, G_2, and J_2 are the CARP.

(8) $\tilde{B}$, $\tilde{F}$, $\tilde{H}$, $\tilde{I}$, $\tilde{K}$, and $\tilde{L}$ are the MA.

(9) If the special cases of the last section do not exist, the MAOR and MARC will not be considered. If they exist, the MAOR and MARC will not be considered. If they exist, the result will be a network in which the MAOR will have L as precedent activity and all MA as subsequent activities; the MARC will have all MA as precedent activities and L' as the subsequent activity.

The consideration of this example, in which some symetric activities appear, will not restrict the conclusions of this research. Examples in which this condition is not observed may be presented and solved by the guidelines herein enunciated.

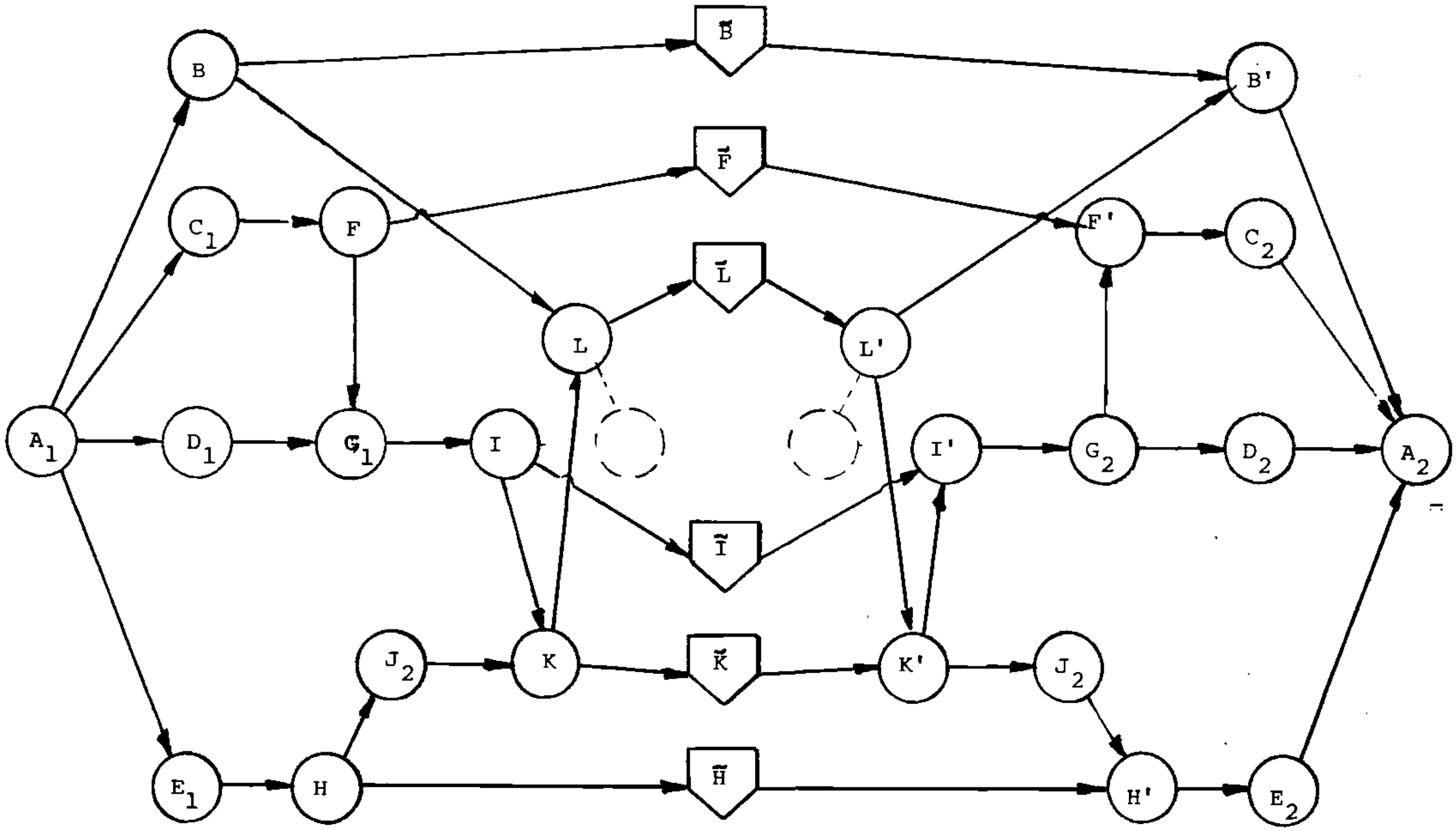


Figure 4. Theoretical Network of a Project (TNP)

CHAPTER IV

IDENTIFYING THE REPAIRS TO BE INCLUDED IN THE MFN

Conditional Probability of Failure

After designing the TNP for a given system, we are ready to take the next step in the process of identifying the Most Favorable Network. In this step, we establish, for each part, the probability that failure occurred in that part of the system, given that the system failed at time t.

It is inherent that every part of the system has a hazard rate of failure, $z_i(t)$, associated with it. The hazard rate function could be any of those described in Chapter II or another function that could be described for a specific case.

To improve the understanding of this presentation some specific conditions will be considered to hold. These will not make the proposed approach lose its generality. These conditions are the following:

(1) The TNP of Figure 4 is the basis of all examples.

(2) After maintenance has been completed the parts

will recover the same characteristics they had when new.

(3) The hazard rates of failure $z_i(t)$ for the mode or part i are functions of time, and they assume values given in Table 1.

Table 1. Hazard Rates of Failure for Some Number of Operating Hours.

	Hazard Rate of Failure, $z_i(t)$		
Activity	Hours That Part Has Operated		
	10,000	20,000	30,000
B	3.90×10^{-4}	6.10×10^{-4}	7.20×10^{-4}
F	6.30	8.70	19.20
I	2.20	9.80	15.20
H	1.80	6.20	8.80
K	1.20	4.80	20.00
L	1.00	3.00	9.60

The probability that failure occurred in the ith. part (i = 1, 2, ..., 6) given that the system failed at time t is given by

$$A_i(t) = \frac{z_i(t)}{\sum_i z_i(t)} \qquad (8)$$

Thus, by considering that the system has worked, e. g., 10,000, 20,000, and 30,000 hours before the first failure, the $A_i(t)$ values will be given in Table 2. Note that Table 1 will remain valid for the system, but Table 2 will have to be recalculated after each repair.

Table 2. $A_i(t)$ Values for Some Number of Operating Hours

Activity	Hours the System Has Operated		
	10,000	20,000	30,000
B	0.237	0.158	0.090
F	0.385	0.225	0.240
I	0.134	0.253	0.190
H	0.110	0.161	0.110
K	0.073	0.125	0.250
L	0.061	0.078	0.120

Measure of Effectiveness for Evaluation of Alternative Starting Networks.

General

After calculating the $z_i(t)$ and $A_i(t)$ values for each part at time t, the next step in the process of identifying the MFN is to establish a measure that will indicate which starting maintenance network will be the best one for the project under consideration. This measure shall be determined for each particular system condition and shall be considered as a problem of optimal decision making.

Optimal decision making implies the existence of criteria that can be used to determine when an optimal decision has been reached in terms of benefits received in relation to the costs involved. A decision is always a choice among alternatives, each of which will lead to a specific outcome, although the exact outcome is not generally known to the decision maker, nor will it necessarily be a desirable outcome.

In some cases whether a desirable outcome or some other outcome occurs as a result of the selected action, there will not be a significantly different action. Often however, the effect of an undesirable outcome may have a very large penalty attached to it. In such a case the decision maker would like to have a high confidence that such an undesirable outcome will not occur.

One would like, whenever possible, to use a logical quantitative analysis in order to predict the possible outcomes. However, analytical methods may fail or may be deficient. Then it is necessary to fall back on experience, judgement, intuition, and one's individual set of values. Some people depend upon pure chance or on their own prejudices to arrive at a decision. However, a more rational, consistent approach to decision making is highly desirous in order to optimize the value of the outcome.

Thus the basic decision making problem is one of choosing an action from a field of available actions. If one is indifferent in this choice, then the decision is trivial. Therefore, regardless of which action is chosen one has given equal weight or value to the outcomes of such a decision.

A much more realistic rule of decision making is choosing that action which will result in the most favorable outcome, or which one may expect will result in the most favorable outcome, or perhaps that action which will result in the least risk, loss, cost, or any other undesirable effect.

Thinking in these terms, one has to choose a starting maintenance network, one which, it is expected will result in <u>the most favorable network</u>. The Most Favorable Network will be used herein to identify the starting maintenance network which has the least expected cost of changes due to inspection outcomes.

<u>The Elements of Decision Making Theory</u>

Let us define the following decision situations: let $(A) = (a_1, a_2, \ldots, a_i, \ldots, a_m)$ be a set of m possible alternative actions, and $(S) = (s_1, s_2, \ldots, s_j, \ldots, s_n)$

be a set of n possible states of nature occurring with a corresponding probability $(P) = (p_1, p_2, \ldots, p_j, \ldots, p_n)$ where $0 \leq p_j \leq 1$, and $\sum_j p_j = 1$.

Then, each action-state-of-nature pair (a_i, s_j) will have associated with it an outcome O_{ij} with an associated value u_{ij} representing the gain or utility to the decision maker of choosing the action a_i if the state of nature turns to be s_j.

The decision situation may then be shown as a decision matrix as illustrated in Figure 5. For a decision problem under certainty, this matrix is reduced to a single column. In that case, u_{ij} represents the value of the ith. action, as state s_j is known to exist. It is obvious that we should choose that action a_i which produces the highest utility.

States	s_1	s_2	$\cdots$ s_j	$\cdots$ s_n
Probability	p_1	p_2	$\cdots$ p_j	$\cdots$ p_n
Actions		Utility		
a_1	u_{11}	u_{12}	$\cdots$ u_{1j}	$\cdots$ u_{1n}
a_2	u_{21}	u_{22}	$\cdots$ u_{2j}	$\cdots$ u_{2n}
.	.	.	... .	... .
.	.	.	... .	... .
.	.	.	... .	... .
a_i	u_{i1}	u_{i2}	$\cdots$ u_{ij}	$\cdots$ u_{in}
.	.	.	... .	... .
.	.	.	... .	... .
.	.	.	... .	... .
a_m	u_{m1}	u_{m2}	$\cdots$ u_{mj}	$\cdots$ u_{mn}

Figure 5. Theoretical Decision Matrix

In choosing a starting maintenance network there is a problem of decision under risk. In this case, no single value of utility can be associated with a given action, but rather, one can determine the expected utility associated with that action by calculating $\sum_{j=1}^{n} u_{ij}p_j$ for that particular a_i. The utility measure is the cost of converting the starting maintenance network implied by a_i to the network which is exactly designed for dealing with the state s_j.

The Maintenance Network Problem and a Further Look Into The Decision Elements.

Here it seems appropriate to review several points discussed previously. We have seen that a decision problem, as exemplified by the decision matrix of Figure 5, contains the following elements: (a) alternative actions, (b) states of nature, (c) probabilities assigned to each state of nature, (d) outcomes, each of which has a certain utility to the decision maker, and (e) some rule of decision making by which the information in the decision matrix will be processed and an action chosen.

Let us examine these elements to determine how one may utilize them, along with the elements of the maintenance network, to structure a decision problem.

Alternative Actions. There is an almost endless of actions one could consider in structuring his decision problem. However, in any decision problem, the decision maker must list what he considers to be meaningful actions on a mutually exclusive basis. Generally this is not too difficult.

We call alternative actions for a maintenance project all the starting maintenance networks that result from all possible combination of failing parts (BA). Consequently, there will be for each combination a network associated with it so that the parts of each combination can be repaired. In this sense, there will be m alternative actions or starting maintenance networks for a project, such that

$$m = \sum_{k=1}^{n} C_k^n \tag{9}$$

in which

n = no. of parts (BA)

k = no. of possible repairing parts.

Let us represent the starting maintenance networks by N_i, with $i = 1, 2, \ldots, m$.

For the project in Figure 4, there will be 63 possible starting maintenance networks, because there exist six parts in the system.

<u>States of Nature</u>. Though it is not necessarily desirable to enumerate all possible actions, it is usually necessary to enumerate all states of nature in order not to violate the probability axiom that the sum of all probabilities is 1.

States of nature will be associated with the networks that are obtained by considering the parts of the system one by one failing separately. Consequently, there will be n

states, such that n is the number of parts of the system.

For the project of the Figure 4, there will be six states of nature. These six states are the following: B, H, F, I, K, and L. Consequently, for each state there will be a network associated with it such that the failed part will be repaired.

Utility Values. If actions and states of nature are enumerated, then one should be able to say something about the outcomes of each action-state pair (N_i, s_j). To each of these pairs some utility values must be assigned.

In many cases the utility values may only be represented by a range of values, or by a probability distribution, or some functional relationship (usually non-linear and perhaps discontinuous). It may not even necessarily be a number.

Before determining the utility values for the decision model, let us review several points. We have seen that the alternative actions and states of nature of the decision matrix of Figure 5 have already been determined. We have seen that the states of nature may be optimally dealt with by networks obtained by considering the failing parts one by one. We have also seen that the alternative actions N_i select possible starting networks. Then, if one action is chosen, there will be a network associated with it.

On the other hand, each combination of an act and a state, pair (N_i, s_j), has a payoff associated with it. This payoff represents the cost associated with the modifications that will have to be done in the network N_i, if this network differs from the network associated with the state s_j. This cost should reflect the activity deletion effect, as well as the activity addition effect between those networks. Problems related with criticality, resource leveling, and re-scheduling should be pointed out, if possible. To simplify the analysis, let us suppose that this cost is directly related to the number of changes necessary to go from the chosen network to the pointed state. The changes represent the number of activities added to and/or subtracted from the network N_i in order to achieve the network associated with the state s_j. Then,

$$u_{ij} = - k\, n_{ij} \tag{10}$$

in which

u_{ij} = utility value of the pair (N_i, s_j)

n_{ij} = number of changes required

k = a proportionality constant.

Since we are going to establish a relative measure among alternative actions, the u_{ij} values in the decision matrix of Figure 5 can be changed by the $(-n_{ij})$ values.

Probability of the State of Nature. The problem of assigning probabilities is often one of the more difficult aspects of the decision problem at the outset. Here is the situation in which past experience and knowledge about events and the environment should be brought to bear on the problem. Human judgement is very important.

If we claim to have no information whatsoever about the states of nature (problem environment), we obviously cannot assign different probabilities to them. However, even if in the state of "complete ignorance", we can always start the decision problem by using the principle of insufficient reason and by assigning equal probabilities to all states of nature, or by using whatever information we have from the past, personal beliefs, or from the judgements of experts in order to assign subjective probabilites to the various states. However in this research the probability of state s_j at time t is exactly the values $A_j(t)$ calculated previously.

Considering the assumptions made up until now, the decision matrix will have the format of Figure 6.

States	s_1	...	s_j	...	s_n
Probabilities	A_1	...	A_j	...	A_n
Actions			Utilities		
N_1	$-n_{11}$	...	$-n_{1j}$	...	$-n_{1n}$
.	.	...	.	...	.
.	.	...	.	...	.
.	.	...	.	...	.
N_m	$-n_{m1}$	...	$-n_{mj}$	...	$-n_{mn}$

Fig. 6. Decision Matrix for a Maintenance Project.

The Rule of Decision Making. Most decision theorists, e.g., C. Fishburn, H. Raiffa, today generally agree that the one rule of decision making which is most logically consistent is that of maximizing expected utility. The general statement for the expected utility stated earlier and the assumption of maximizing it may be stated in the following form for a maintenance project:

$$\max_i E\ (N_i) = \max_i \sum_j (-n_{ij})\ A_j \tag{11}$$

for $i = 1, 2, \ldots, m$

$j = 1, 2, \ldots, n$

The starting maintenance network N_i with the maximal expected utility will be the MFN for the project. The procedures for creating the MFN from the TNP will be given in the following chapter.

A Procedure for Calculating the Maximal Expected Utility

In searching for the maximal expected utility for a given system failure it is not necessary to calculate all the expected utilities of all the alternative actions. The search can be done in stages, i.e., the expected utilities of the networks associated with the combinations of one and of two of the possibly failed parts are calculated. If the maximal expected utility is associated with a network that includes the repair of only one part, the search will stop. But, if the maximal expected utility is associated with a network that includes the repair of two parts, then the search will be extended to those networks in which three parts can be repaired, and so forth.

Example 1

Let Figure 4 be the TNP under the maintenance process in which the basic activities are B, F, H, I, K, and L. Let us suppose that the system has failed at time t - 10,000 hours. By using the values of Table 1 and

Table 2 we have the following:

(1) $A_j(t)$ values

$A_B = 0.237$; $A_F = 0.385$; $A_H = 0.110$

$A_I = 0.134$; $A_K = 0.073$; $A_L = 0.061$

(2) The states of nature

B, F, H, I, K, and L

(3) Probabilities of the states of nature

$p_j = A_j$ for j = B, F, I, H, K, and L.

(4) Possible starting maintenance networks.

There are 63 possible starting maintenance networks one for each of the following combination of parts whose repair is to be included:

B, F, H, I, K, L

BF, BH, BI, ..., KL

BFH, BFI, BFK, ..., IKL

BFHI, BFHK, BFHL, ..., HIKL

BFHIK, BFHIL, ..., FHIKL, and BFHIKL.

(5) Table 3. Numerical Decision Matrix at t = 10,000 Hours

States	B	F	H	I	K	L	$E(N_i)$
Probab.	0.237	0.385	0.110	0.134	0.073	0.061	
Actions			Utilities				
B	0	- 8	- 8	-14	-22	- 22	- 8.80
F	- 8	0	-10	- 8	-20	-20	- 6.76*
H	- 8	-10	0	-16	-16	-20	-10.28
I	-14	- 8	-16	0	-10	-14	-10.76
K	-22	-16	-16	-10	0	- 6	-14.91
L	-22	-20	-20	-14	- 6	0	-17.43
BF	- 5	- 3	-13	-11	-19	-19	- 7.77
BH	- 5	-13	- 3	-19	-19	-19	-11.59
BI	-11	-11	-19	- 3	-13	-13	-11.10
BK	-19	-19	-19	-13	- 3	- 5	-15.20
BL	-21	-21	-21	-15	- 7	- 1	-17.98
FH	-13	- 5	- 5	-13	-13	-17	-10.40
FI	-15	- 7	-17	- 1	-11	-15	-10.26
FK	-23	-15	-17	-11	- 1	- 7	-20.08
FL	-23	-19	-21	-15	- 7	- 1	-17.68
HI	-19	-13	-11	- 5	- 7	-11	-12.56
HK	-23	-17	-15	-11	- 1	- 7	-15.58
HL	-23	-21	-19	-15	- 7	- 1	-18.21
IK	-23	-17	-17	- 9	- 1	- 7	-15.56
IL	-23	-21	-21	-13	- 7	- 1	-18.17
KL	-25	-21	-21	-15	- 5	- 1	-18.74

Based on the results of this matrix one concludes that the starting maintenance network at t = 10,000 hours which accounts for the repair of the part F is the MFN.

Example 2

Let Figure 4 be the TNP under the maintenance process in which the basic activities are B, H, K, I, F and L. Let us suppose that the system has failed at time t = 30,000 hours. By using the values of Table 1 and Table 2 we have the following:

(1) $A_j(t)$ values

$A_B = 0.090$; $A_F = 0.240$; $A_H = 0.110$

$A_I = 0.190$; $A_K = 0.250$; $A_L = 0.120$

(2) The states of nature

B, F, H, I, K, and L

(3) Probabilities of the states of nature

$p_j = A_j$ for j = B, F, I, H, K, and L

(4) Alternative actions or possible starting maintenance networks

There are 63 possible starting maintenance networks one for each of the following combination of parts whose repair is to be included:

B, F, H, I, K, L

BF, BH, BI, ..., KL

BFH, BFI, BFK, ..., IKL

BFHI, BFHK, BFHL, ..., HIKL

BFHIK, BFHIL, ..., FHIKL

BFHIKL.

(5) Table 4. Numerical Decision Matrix at t = 30,000 Hours

States	B	F	H	I	K	L	$E(N_i)$
Probab.	0.090	0.240	0.110	0.190	0.250	0.120	
Actions			Utilities				
B	0	- 8	- 8	-14	-22	-22	-14.20
F	- 8	0	-10	- 8	-20	-20	-10.74
H	- 8	-10	0	-16	-16	-20	-12.56
I	-14	- 8	-16	0	-10	-14	- 9.12*
K	-22	-16	-16	-10	0	- 6	-10.20
L	-22	-20	-20	-14	- 6	0	-14.14
BF	- 5	- 3	-13	-11	-19	-19	-11.72
BH	- 5	-13	- 3	-19	-19	-19	-14.54
BI	-11	-11	-19	- 3	-13	-13	-11.10
BK	-19	-19	-19	-13	- 3	- 5	-12.21
BL	-21	-21	-21	-15	- 7	- 1	-13.96
FH	-13	- 5	- 5	-13	-13	-17	-11.68
FI	-15	- 7	-17	- 1	-11	-15	- 9.64
FK	-23	-15	-17	-11	- 1	- 7	-11.72
FL	-23	-19	-21	-15	- 7	- 1	-14.66
HI	-19	-13	-11	- 5	- 7	-11	-10.06
HK	-23	-17	-15	-11	- 1	- 7	-12.89
HL	-23	-21	-19	-15	- 7	- 1	-14.92
IK	-23	-17	-17	- 9	- 1	- 7	-11.82
IL	-23	-21	-21	-13	- 7	- 1	-14.76
KL	-25	-21	-21	-15	- 5	- 1	-14.82

Based on the results of this matrix one concludes that the MFN should show the repair of the part I.

CHAPTER V

CREATING THE MOST FAVORABLE NETWORK (MFN)

Objectives

In this chapter the steps to develop the MFN will be established, and the format of the MFN will also be identified. The procedures involves converting the TNP into the MFN, so that an initial schedule can be designed. The MFN may contain one or more MA. If it contains one MA, it may still possibly have to be changed when the inspection portion of the MA is carried out because this inspection may show that that part has not failed. If the MFN contains more than one activity, it will certainly require change as the inspection stage is reached and true identity of the failed part is determined.

Steps in the Selection of the MFN

The following steps must be considered for drawing up the Most Favorable Network:

Step 1 : Consider the TNP with all activities: BA, IA, MA, CAOP, CARP, CACP, START and END activities.

<u>Step 2</u> : Identify the hazard rate of failure for each part (and hence each MA) for the point in time at which the failure occurred.

<u>Step 3</u> : Determine, for each part, the probability, $A_i(t)$, that the system failure is due to the failure of that part.

<u>Step 4</u> : Select the parts whose repair should be accounted for in the MFN. This involves determining the maximum utility of selected possibilities as discussed in Chapter IV.

<u>Step 5</u> : Refer to the MA associated with each part so chosen for inclusion in the MFN as a dangerous activity (DA); otherwise, an MA will be considered a good activity (GA).

<u>Step 6</u> : In the TNP, just assign zero duration to or eliminate all activities associated with parts not selected for consideration, i.e., all GA and associated BA, IA, etc.

<u>Step 7</u> : Use the middle activities in the following manner: MAOR and MARC are dummy activities and are used to eliminate some doubts that may exist related to the precedence relationship problem, e.g.,

(1) The MAOR should be used when:

(a) It is necessary to have an end point for the OP and/or a starting point for the RP.

(b) There exist some activities parallel to all activities of the OP and/or RP, in order to maintain the precedence relationships among the OP and RP.

(2) The MARC should be used when:

(a) It is necessary to have an end point for the RP and/or a starting point for the CP.

(b) There exist activities parallel to all activities of RP and/or CP in order to preserve the precedence relationships among the RP and CP.

Step 8 : This step ought to be followed when the maintenance project is being developed. At the end of each BA establish an inspection and decision making point, i.e., at each BA the programmer decides if the MA associated with this BA is actually required, for the inspection makes it obvious that either:

(1) The part has to be repaired.

(2) The part does not have to be repaired.

In case (1) all the subsequent activities related to the BA will hold. However in case (2) the subsequent activities related to the BA in the RP will no longer exist. Therefore, to keep the stability of the MFN, one will give time zero for the activities in the RP, and one will maintain only the activities necessary to carry the project to an end. Acting in such a way, we automatically update the network.

Definition of the MFN

The network that is obtained by applying Steps 1-7 as formulated in the preceding section is said to be The Most Favorable Network for the maintenance project.

Examples of MFN

The format of the MFN of a maintenance project as well as the transition from the TNP to the MFN will be illustrated by two examples for different situations.

Let us consider the following assumptions:

(1) The TNP of Figure 4.

(2) The data from Table 1, Table 2, Table 3 and Table 4.

If the system has worked 10,000 or 30,000 hours before failing, the DA will be given in Table 5 by using the results of Table 3 and Table 4.

Table 5. List of Dangerous Activities

Hours	Dangerous Activities
10,000	$\tilde{F}$
30,000	$\tilde{I}$

Thus, following the steps for drawing the MFN, one will get the networks given in Figure 7 and Figure 8 for respectively 10,000 and 30,000 operating hours. These networks have the following characteristics:

(1) MFN characteristics at t = 10,000 hours (Fig. 7)

(a) Activity START .... A_1

(b) Activity END .... A_2

(c) Dangerous activity .... $\tilde{F}$

(d) Basic Activity .... F

(e) CAOP .... C_1

(f) Image activity .... F'

(g) CACG .... C_2

(2) MFN characteristics at t = 30,000 hours (Fig. 8)

(a) Activity START .... A_1

(b) Activity END .... A_2

(c) Dangerous activity .... $\tilde{I}$

(d) Basic activity I

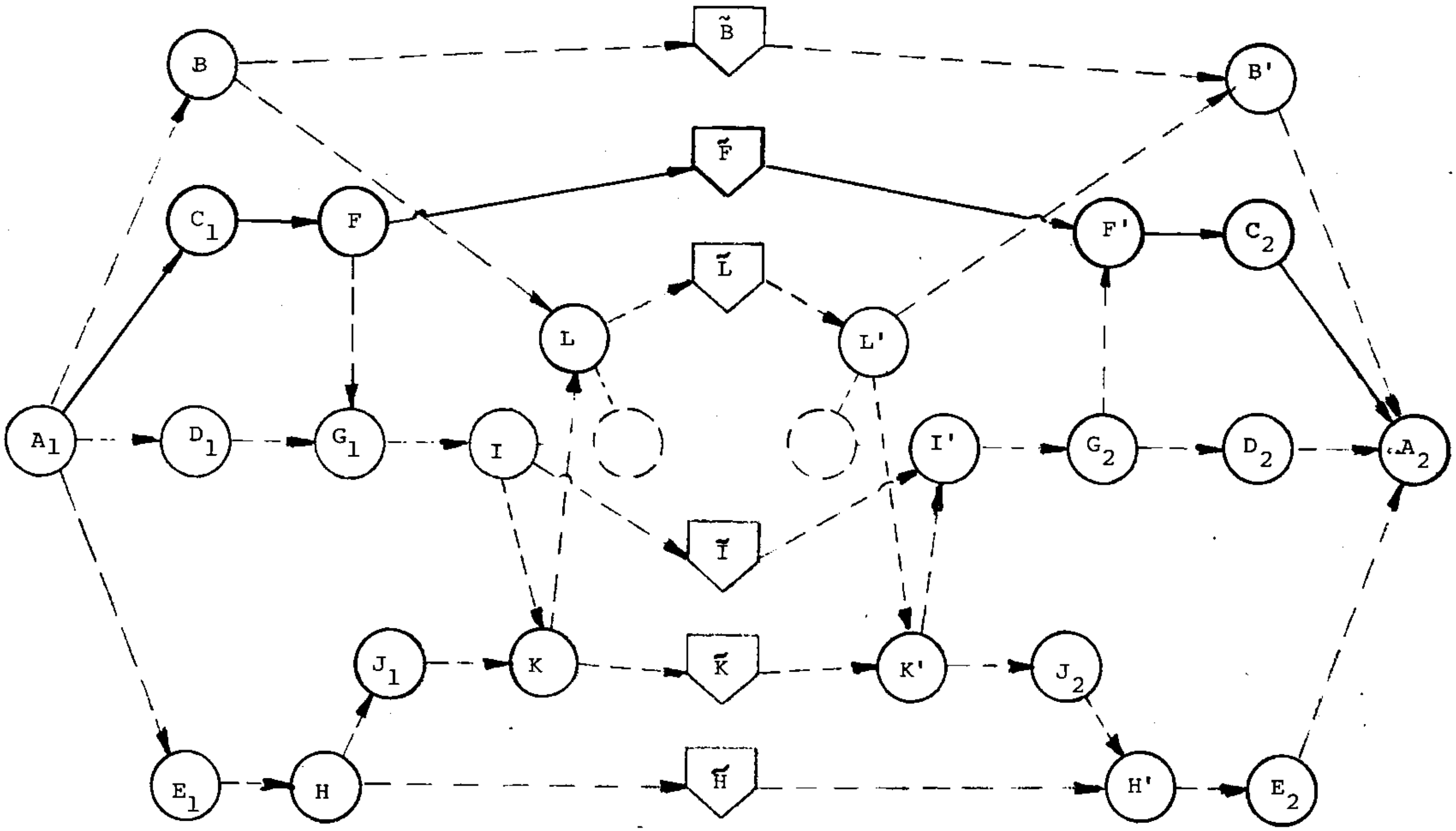


Figure 7. MFN for 10,000 Hours

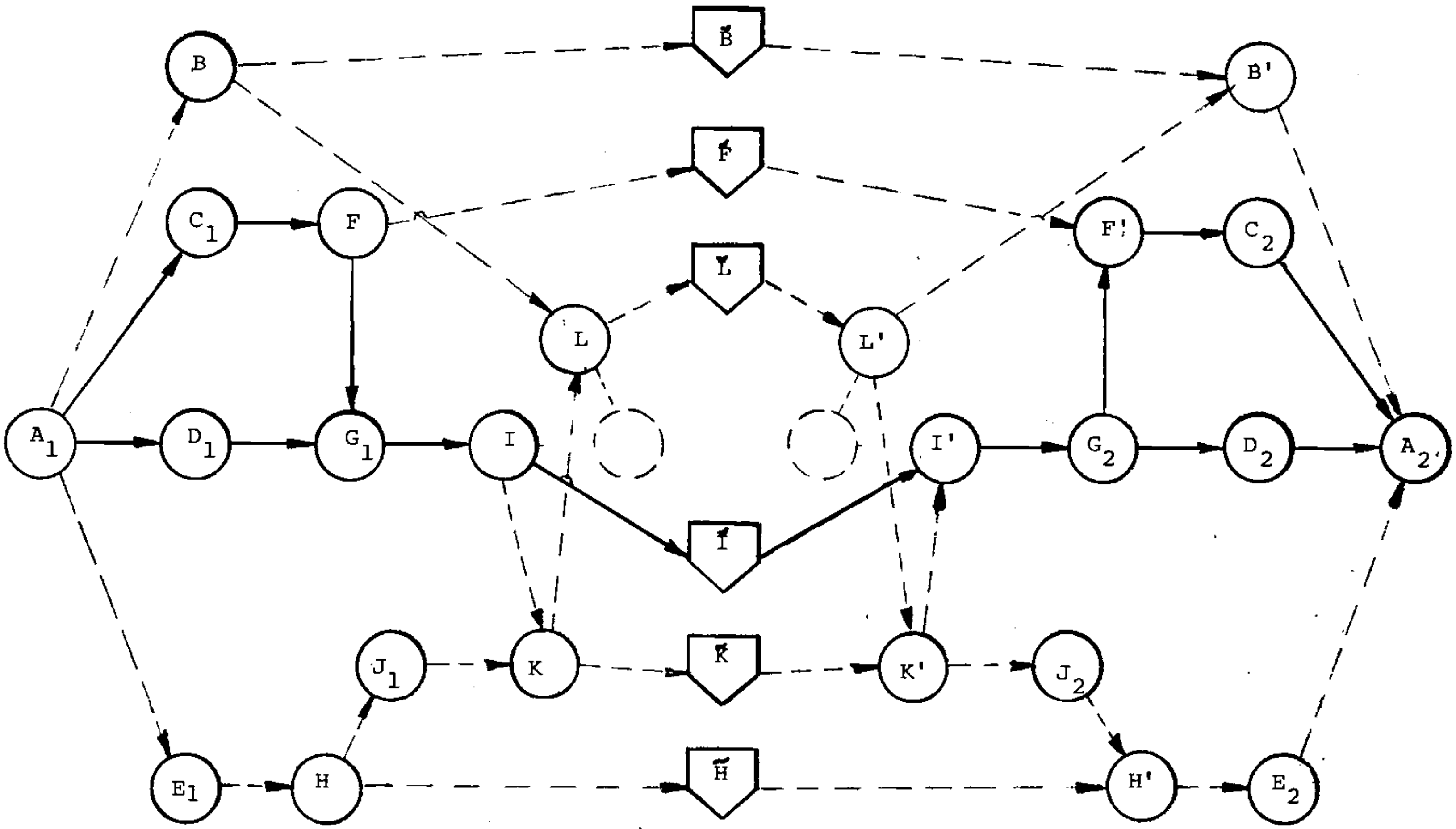


Figure 8. MFN for 30,000 Hours

(e) CAOP .... F, C_1, D_1, G_1

(f) Image activity .... I'

(g) CACP .... F', C_2, D_2, G_2

Special Example

Let us consider that the system has the TNP of Figure 4, has worked 30,000 hours and has the characteristics given by Table 1, Table 2, and Table 4. Furthermore let us suppose that the TNP has the following additional activities:

(1) Activity X , CAOP, parallel to OP

(2) Activity Y , CACP, parallel to CP

(3) Activity Z , CARP, parallel to RP.

These activities could represent some administrative actions that should be taken during the three phases of a maintenance project. By doing these considerations, the MFN will be that shown in Figure 9, in which the middle activities exist.

If, instead of X, Y, and Z, one considers only the activity X in Figure 9, the additional precedence relationships among the MA and the activities in CP will no longer exist. This is shown in Figure 10.

If, instead of X, Y, and Z, one considers only the

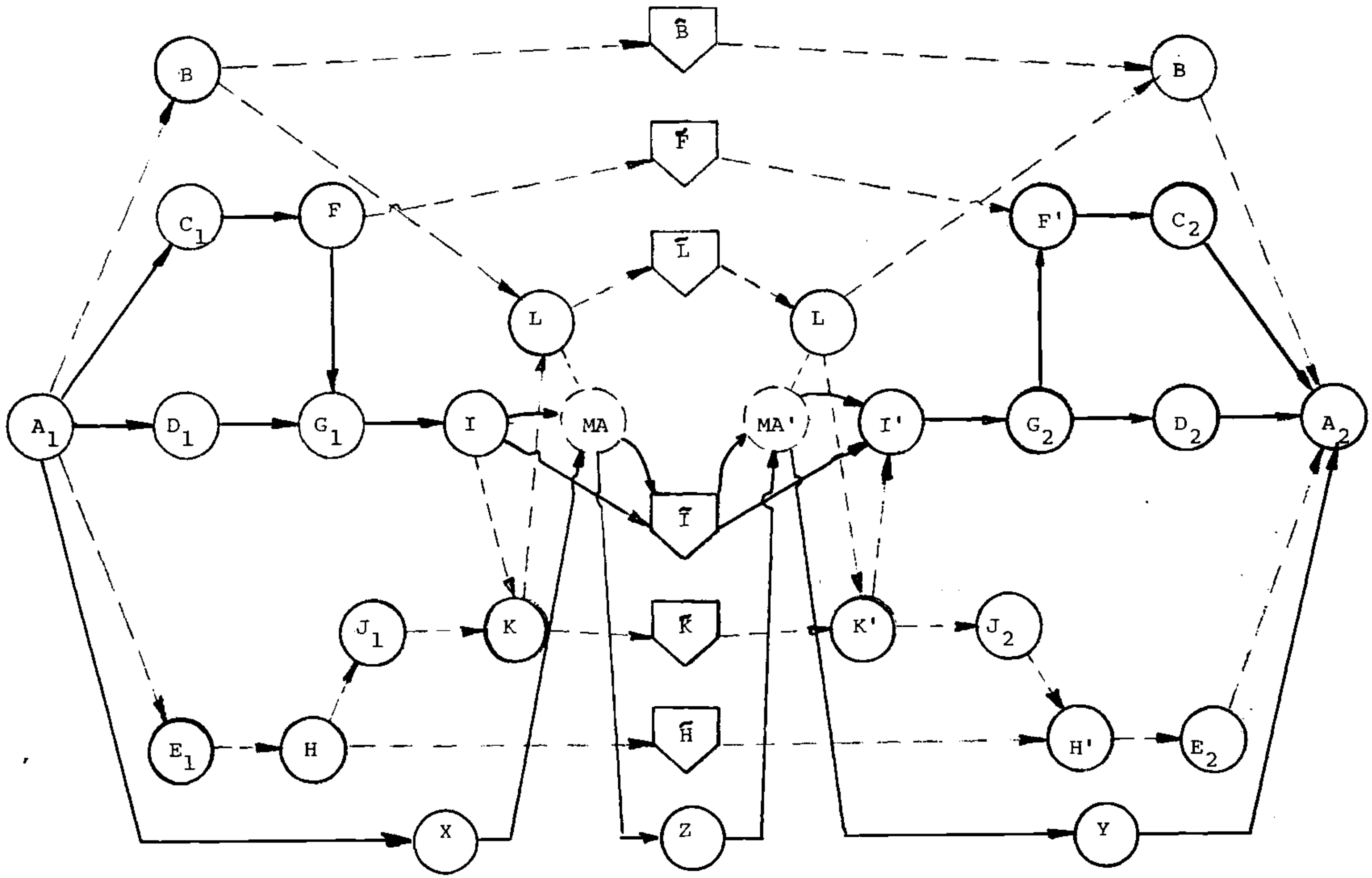


Figure 9. MFN at 30,000 Hours for a TNP with Activities X, Y and Z

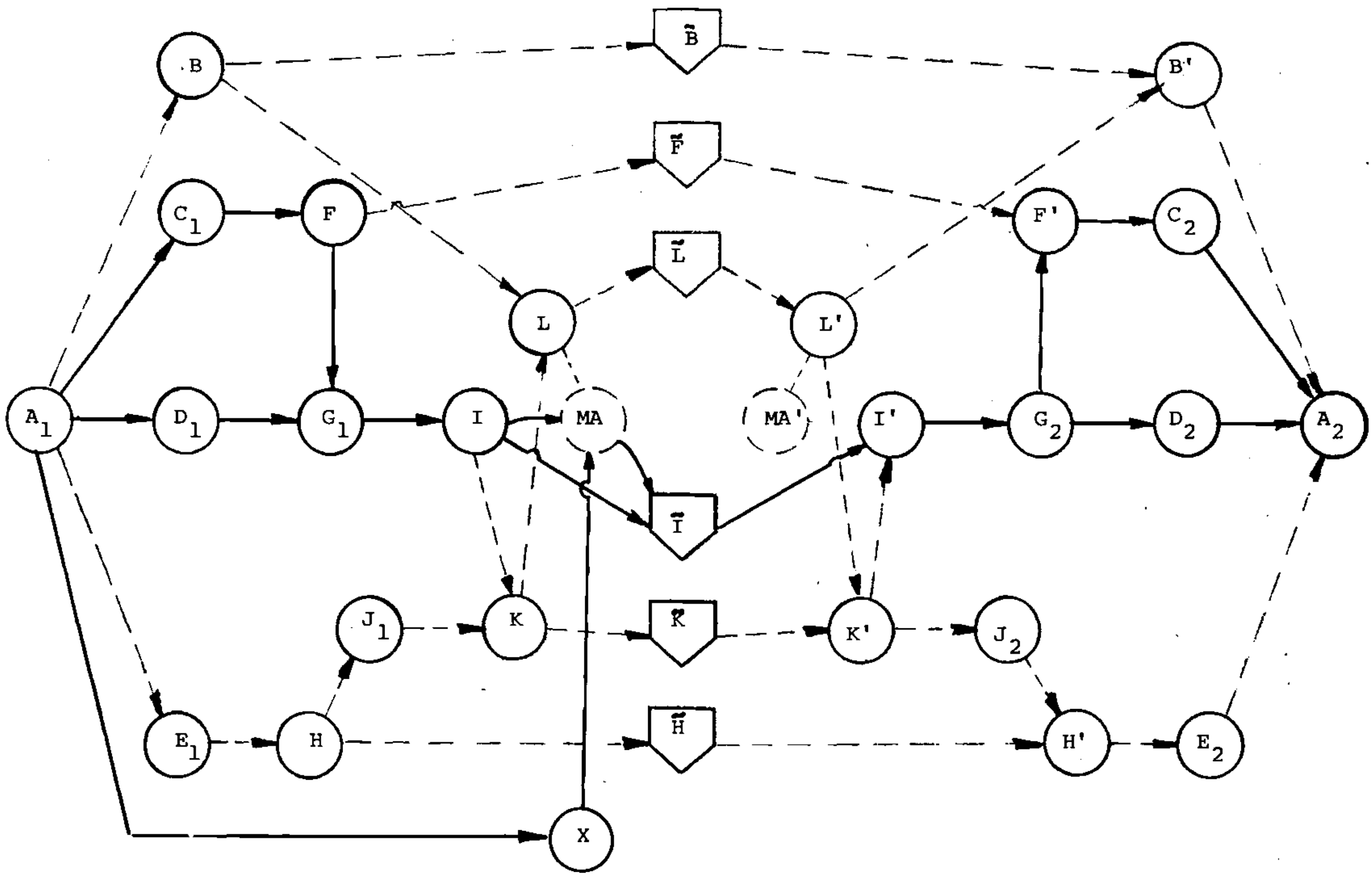


Figure 10. MFN at 30,000 Hours for a TNP with Activity X

activity Y in Figure 9, the additional precedence relationships among the MA and the OP will no longer exist. This can be seen in Figure 11.

If, instead of X, Y, and Z, one considers only the activity Z and/or the activities X and Y, the additional relationships of the Figure 9 will exist.

Comments

With these examples we have a good idea of the meaning of the MFN as well as its relationship to the Theoretical Network of the Project.

These examples reinforce the author's ideas concerning the application of reliability theory to the problems of drawing a network for short-run maintenance projects.

It seems to the author that the MFN, for each situation, is "closer" to the correct network of the system than any one that could be constructed without the decision theory and statistical bases.

Time Evaluation

To complete the relation between reliability theory and the MFN for short-run maintenance projects, one will associate the concepts of maintainability to the MA as well as their down-times.

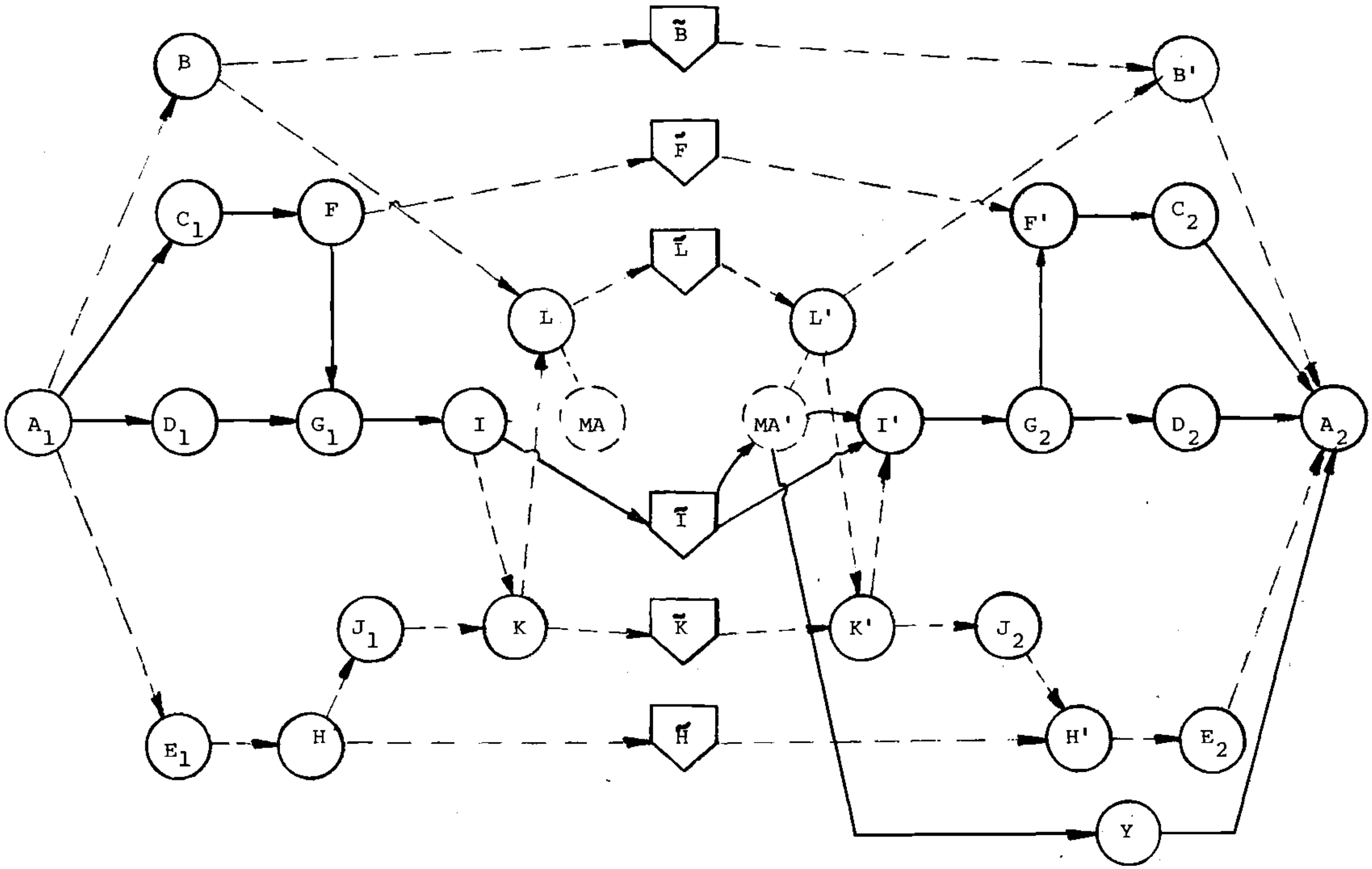


Figure 11. MFN at 30,000 Hours for a TNP with Activity Y

Obviously there exists a probability associated with each estimated down-time. Maintainability theory shows the existence of some statistical distributions of the down-time such as Normal, Log-Normal, and Exponential distributions. Depending on the case, these distributions give the best information about down-times. The distributions suggested by maintainability theory are more realistic to explain the down-time than the Beta-distribution, which is normally used in PERT-CPM theory.

Network Solution and Its Updating

With the MFN drawn, we will be able to determine and complete with more accuracy the last two phases of the network based project management, i.e., the scheduling and control phases.

Obviously, with the development of the maintenance operation, the network may be changed to state the actual situation. However, the probability of extensive changes as well as of excessive costs associated with the planning phase tends to be lower than if the MFN has not been studied. It seems to the author that the more accurate the $A_i(t)$ determination and the more accurate the utility evaluation, the faster and cheaper will be the development of the real network.

CHAPTER VI

EXTENSIONS OF THE MFN APPROACH TO OTHER TYPES OF MAINTENANCE PROCESSES

Block Diagram

With some additional considerations it is possible to apply the same concepts seen up to now to other maintenance problems. The block diagram in Figure 12 shows the cases in which it is possible to apply and extend the concepts herein proposed.

Analysing the possible paths in the block diagram in Figure 12, one may note that the approach was developed based on the case identified by the path 1.2.3.4.5.6 . However, the other cases in the diagram may be planned by using the approach here developed.

In the cases 1.2.3.4'.5".7, 1.2.3.4'.7 , and 1.A.B.C , the programmer knows what is going to happen, but the information about the parts that ought to be replaced is given by the reliability theory. Furthermore, in these cases the MFN will be the real network for the maintenance process under consideration.

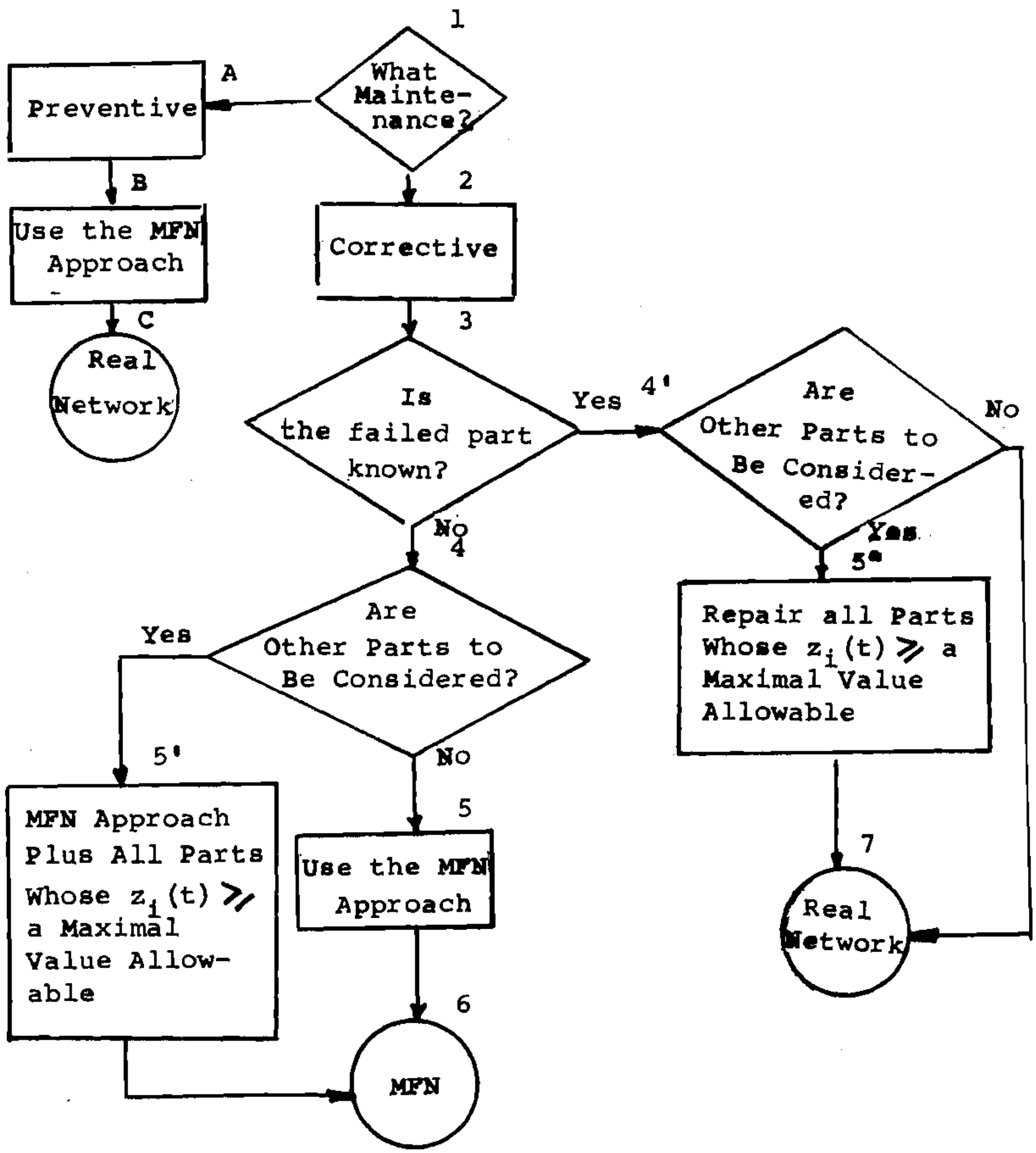


Fig. 12. Relationship Between the MFN Approach and the Types of Maintenance - Extensions.

In the case 1.2.3.4.5'.6 we also get a MFN network.

The procedures here developed may be applied to almost all cases of corrective and preventive maintenance processes.

CHAPTER VII

CONCLUSIONS

Guide for Planning Maintenance Projects

As a consequence of this research one can establish a general guide that should be followed for the planning of maintenance projects. This procedure can be divided into the following steps:

Step 1 : Draw the theoretical network of the project (TNP) following the steps given herein.

Step 2 : Determine the hazard rate of failure for each part, for the point in time at which the failure occurred.

Step 3 : Determine, for each part, the probability, $A_i(t)$, that the system failure is due to the failure of that part.

Step 4 : Select the parts whose repair should be accounted for in the MFN. This involves determining the maximum utility of the selected possibilities as discussed in Chapter IV.

Step 5 : Consider as "key points" the MA, BA, and IA activities that are associated with these parts.

Step 6 : Draw the Most Favorable Network using the procedures of Chapter V.

Step 7 : Establish the down-times of the maintenance activities using the maintainability concepts.

Step 8 : Proceed with the last two phases of the network-based project management, i.e., the scheduling and control phases.

Step 9 : If the number of activities in the project is not excessive, use one of the mechanical scheduling and control devices that have been designed. These devices are designed so that the activities can be removed and replaced in different periods of time and the precedence relationships that connect activities can be maintained. With these approaches, rapid adjustment of the plan and/or schedule is possible. Therefore, the application of these devices on the MFN makes the task of achieving the real network for short-run maintenance projects easier.

Step 10 : In all phases that require calculations, computers may be used, since the programs are relatively simple. Remote consoles provide a possibility for holding to a minimum the lag between the acquisition of new knowledge of the activities involved in the project and the creation of updated schedules for the revised project.

Final Conclusions

It seems to the author that the possibility of application of the reliability theory as well as PERT-CPM theory in designing a maintenance network with practical value has been demonstrated.

The author has applied the concepts of his idea to a theoretical example and to some arbitrary $z_i(t)$ functions. However, as one can observe, other $z_i(t)$ as well as practical examples could have been used. In the Appendix A practical data are gathered and the approach applied.

The main problem in obtaining the MFN is establishing the hazard rate of failure of the parts. The maintenance system design engineer must have an important part in the identification of these $z_i(t)$. Of course, the network format will depend on the project studied, upon the subsystems associated with $z_i(t)$ and on the accuracy of the hazard rates of failure. Furthermore, the identification of the utility values in the decision matrix is also of great importance in the approach here studied. In this research the utility values were directly related to the number of changes that must be done in a starting maintenance network in order to achieve the real network for the project and the cost involved was supposed constant for each change. However, in a real project the utility should reflect the actual activity addition and deletion costs, and, in a more accurate analysis, the resource leveling and the re-scheduling costs. These are not constant for every activity.

With real and practical applications new rules and concepts may be added to this research and the final results of these applications will yield the complete structure of the approach that should be adopted. However, the author believes that his main approach to solving problems of maintenance planning will contribute the basis for future development in this field.

BIBLIOGRAPHY

1. Archibald, R.D., and Villoria, R.L., Network-Based Management Systems, (PERT-CPM), John Wiley and Sons, Inc., New York, 1967.

2. ARINC Research Corporation, Reliability Engineering, Prentice Hall, Inc., Englewood Cliffs, N.J., 1964.

3. Barlow, R.E., and Proschan, F., Mathematical Theory of Reliability, John Wiley and Sons, Inc., New York, 1965.

4. Battersby, A., Network Analysis for Planning and Scheduling, Saint Martin's Press, New York, 1964.

5. Bazousky, I., Reliability: Theory and Practice, Prentice Hall, Inc., Englewood Cliffs, N.J., 1961

6. Berman, H., "Try CPM to Cut Turnaround Time 20%," Hidrocarbon Processing and Petroleum REFINNER, Jan., 1962, Vol. 41, no. 1.

7. Bigelow, C.G., "Bibliography on Project Planning and Control by Network Analysis: 1959-61," ORSA, Vol. 10, 1962, pp. 728-31.

8. Brooks, K., "How Arrow Diagramming Speeds Up Turnarounds for Cities Services," The Oil and Gas Journal, Jan. 15, 1962, pp. 94-96.

9. Cone, M.W., "CPP for Maintenance of Steam Turbine," Combustion, Vol. 36, July, 1964, pp. 36-43.

10. Davis, D.J., "An Analysis of Some Failure Data," J. Amer. Stat. Assoc., Vol. 47, 1952, pp. 113-50.

11. Feller, W., An Introduction to Probability Theory and Its Applications, Vol. 1, John Wiley and Sons, Inc., New York, 1968.

12. Gall, H.J., "Cut Downtime on Large Maintenance Jobs with CPS," Power Engineering, Vol. 69, July, 1965, pp. 49-51.

13. Hampshire Technical Research Industrial Commerce Service, "A Bibliography of CPM: 1958-65," Southampton, 1966.

14. Hillier, F.S., and Lieberman, G.S., Introduction to Operation Research, Holden Day, Inc., San Francisco, 1968.

15. Horowitz, J., CPS, Management Control Through CPM-PERT, Ronald Press Co, New York, 1967.

16. Iannone, A.L., Management Program Planning and Control with PERT-MOST and LOB, Prentice Hall, Englewood Cliffs, N.J., 1967.

17. Ireson, WG., Reliability Handbook, McGraw Hill Book Co., New York, 1966.

18. Jorgenson, D.W., McCall, J.J., and Radner, R., "Optimal Maintenance of Stochastically Failing Equipment," Rand Report R-437-PR, April, 1966, Rand Corporation, Sta. Monica, Calif.

19. Kemeny, J.G., Snell, J.L., and Thompson, G.L., Introduction to Finite Mathematics, Prentice Hall, Inc., Englewood Cliffs, Inc., N.J., 1966.

20. Kruse, M.E., "Streamline Your Maintenance Planning with CPS," Plant, no. 23, Dec., 1961, pp. 49-51.

21. Lerda, S., "Bibliography on Network-Based Project Planning and Control Techniques: 1962-65," ORSA, Vol. 14, no. 5, 1966, pp. 925-31.

22. Lewis, B.T., Maintenance Management, J. F. Rider, New York, 1963.

23. Moder, J.J., and Phillips, C.R., Project Management with CPM and PERT, Reinhold Publishing Corp., New York, 1967.

24. Morrow, L. C., Maintenance Engineering Handbook, McGraw-Hill Book Co., New York, 1966.

25. Myers, R.H., Wong, K.L., and Gordy, H.M., Reliability Engineering for Electronic Systems, John Wiley and Sons, Inc., New York, 1964.

26. Newbrough, E.T., Effective Maintenance Management, McGraw-Hill Book Co., New York, 1967.

27. Raadt, J.L.V., "Application of CPP to Process Shutdown Maintenance," Techniques of Plant Engineering and Maintenance, Vol. 17, 1966, pp. 239-41.

28. Reeves, E. R., "Fundamental Techniques of CPP," Techniques of Plant Engineering and Maintenance, Vol. 17, 1966, pp. 233-38.

29. Romig, H.G., "PERT-PEP Reliability," National Symposium on Reliability and Quality Control, 8th., 1962, pp. 24-31.

30. Sherwood, P.W., "Computer Impact on Plant Maintenance," Automation, Vol. 18, no. 11, Nov., 1961, pp. 84-86.

31. Shooman, M.L., Probabilistic Reliability: An Engineering Approach, McGraw-Hill Book Co., New York, 1968.

32. Sobczac, T., "Network Planning: Bibliography," Journal of Industrial Engineering, Nov.-Dec., 1962, pp. 507-11.

33. Stires, D.M., and Murphy M.M., Modern Management Methods PERT-CPM, Materials Management Institute, Boston, 1963.

34. Stormont, D.H., "CPP Assures Safe Turnarounds for Marathon Oil," Oil and Gas Journal, Vol. 62, Jan. 13, 1964, pp. 68-71.

35. Wattel, H.L., Bibliography on Network Scheduling and Control Systems (CPM-PERT), Vol. 2, Hofstra University Yearbook of Business, Hempstead, N.Y., 1964.

36. Welker, E.L., "Relationship Between Equipment Reliability, Preventive Maintenance Policy and Operating Costs," ARINC Research Monograph no. 7, ARINC Research Corporation, Wash. D.C., Feb. 13, 1959.

APPENDIX A

In this section a complete example of determining the MFN of a short-run maintenance project will be developed based on a network presented in Gall's article (12). Originally this network plans the overhaul of a turbine-driven boiler feedwater pump. The precedence relationships are given in Table 6 and the network is given in Figure 13. Basically this network will be the TNP, since two additional activities D and F have been included.

Steps to Draw the MFN

(1). The Theoretical Network of the Project is given in Figure 13, in which,

(a) D and F are basic activities

(b) G and O are maintenance activities

(c) K and R are image activities

(d) A, V, X, Y, and Z are activities required in any network.

(2). Hazard Rate of Failure: let us suppose that the $z_i(t)$ of the activities D and F yields the following values at 10,000 and at 20,000 hours: $z_D(10,000) = 6.0\text{x}10^{-4}$; $z_F(10,000) = 4.0\text{x}10^{-4}$; $z_D(20,000) = 9.0\text{x}10^{-4}$;

Table 6. Activities and Precedence Relationship of the TNP.

Activity	Description	Prec.	Subs.
A	Test electric pump	-	B, C, V
B	Remove pump cover	A	D
C	Remove turbine cover	A	F
D	Remove impeller	B	G, H, E
E	Overhaul librification system	D	J
F	Remove turbine	C	N, O
G	Repair impeller	D	I
H	Clean impeller casing	D	K
I	Balance impeller	G	K
J	Fit lower bearings	E	K
K	Replace impeller	I, H, J	L
L	Fit upper bearings	K	M
M	Replace impeller cover	L	W
N	Dress bearing	F	P
O	Repair turbine	F	Q
P	Fit lower bearings	N	R
Q	Balance turbine	O	R
R	Replace turbine	P, Q	S
S	Replace turbine cover	R	T, U
T	Component tests	S	Z
U	Clearance tests	S	Z
V	Overhaul and calibrate gages	A	X
X	Purge all gages and control	V	Y
Y	Replace gages	X	Z
W	Pack pump shafts	M	Z
Z	Operational tests	W, T, U, Y	-

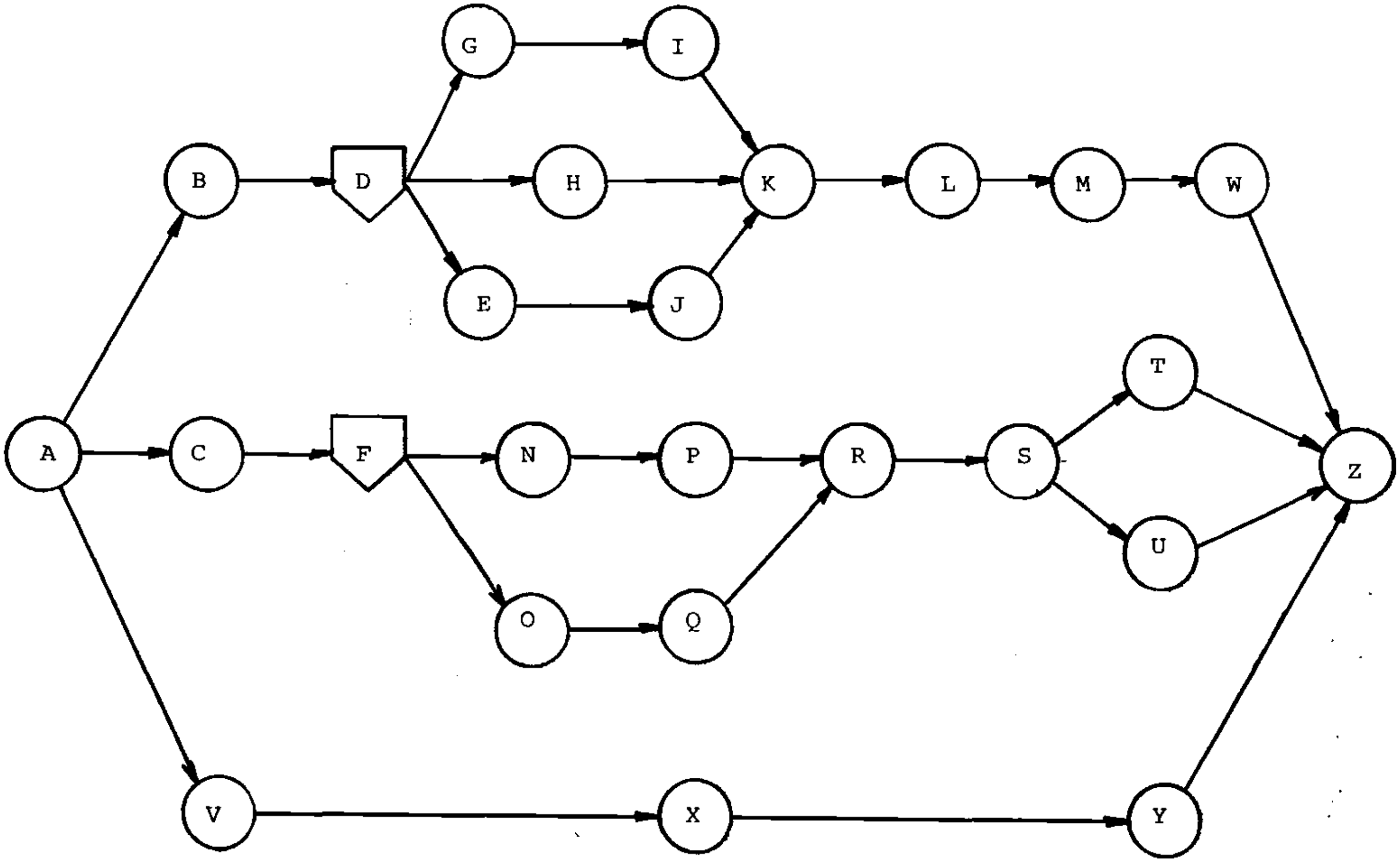


Figure 13. TNP for a Practical Example

$z_F(20,000) = 11.0 \times 10^{-4}$

Consider the system to have failed at 10,000 hours.

(3) $A_i(t)$ values for D and F.

$$A_D = 0.60$$

$$A_F = 0.40$$

(4) The states of nature: the states of nature are obtained by supposing that a part D or F fails separately. For each state there will be an associated maintenance network. These networks are given in Figure 14 and Figure 15 respectively.

(5) Probabilities of the states of nature

$$P_D = A_D = 0.60$$

$$P_F = A_F = 0.40$$

(6) Possible starting maintenance plans: these plans are associated with Figure 13, Figure 14, and Figure 15 by supposing that parts D and F are repaired together or one by one separately. Then we have

Fig. 13 ...supposing D and F being repaired.

Fig. 14 ...supposing D being repaired.

Fig. 15 ...supposing F being repaired.

(7) The decision matrix: this is given in Table 7. As one can observe, the maximum utility for the example is -8.40, and the Most Favorable Network is that one of Figure 14.

Table 7. Numerical Decision Matrix at
t = 10,000 Hours.

States	D	F	Expected Utility
Probability	0.60	0.40	
Actions	Utility		
N_1(D)	0	-21	- 8.40 *
N_2(F)	-21	0	-12.60
N_3(DF)	-10	-11	-10.40

Now consider the system as having failed at 20,000 hours. By following the same steps of the first example, the following decision matrix results:

Table 8. Numerical Decision Matrix at $t = 20{,}000$ Hours.

States	D	F	Expected Utility
Probability	0.45	0.55	
Actions			
N_1(D)	0	-21	-11.55
N_2(F)	-21	0	- 9.45 *
N_3(DF)	-10	-11	-10.55

We conclude that the network of Fig. 15 is the MFN.

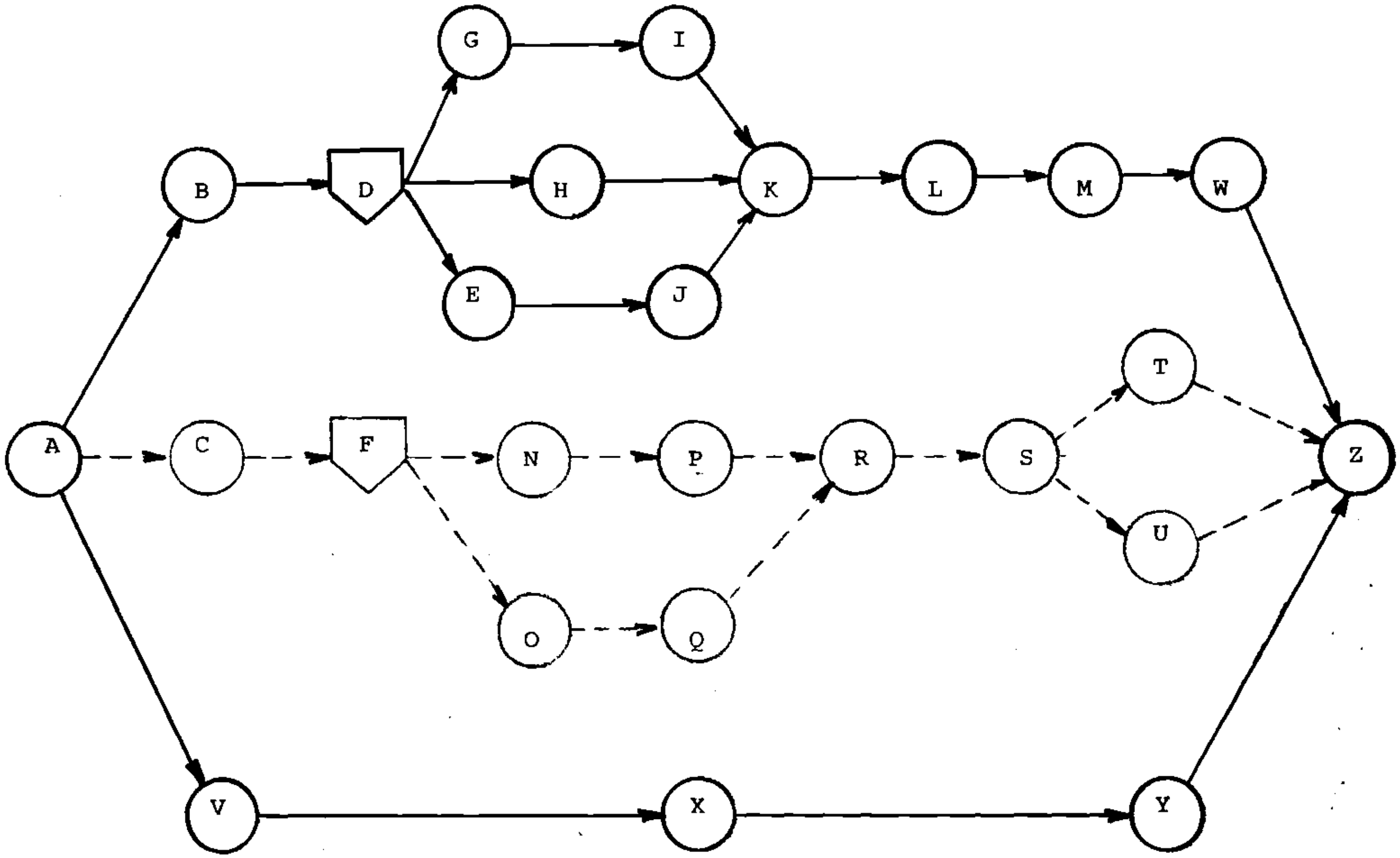


Figure 14. MFN at 10,000 Hours for a Practical Example

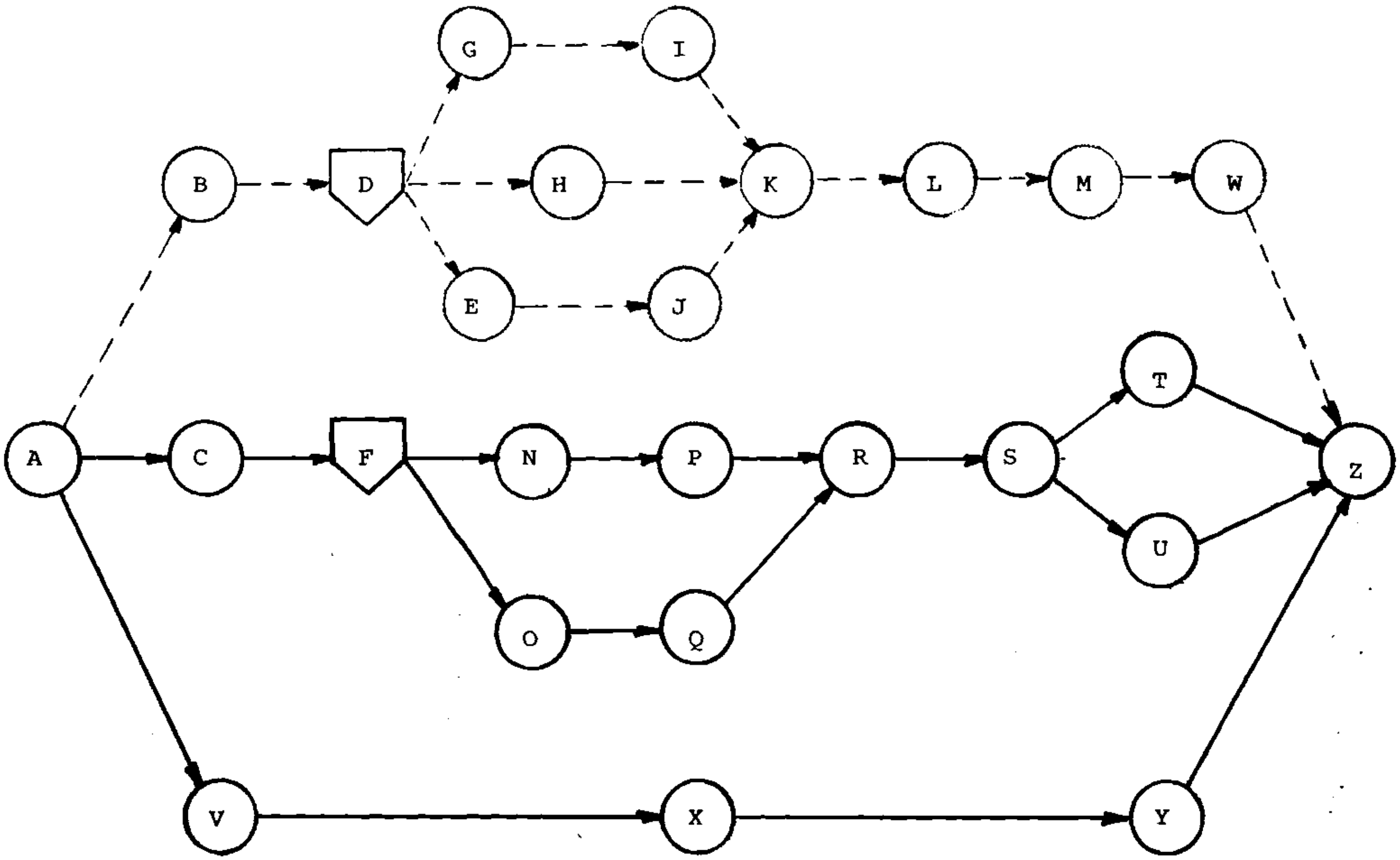


Figure 15. MFN at 20,000 Hours for a Practical Example